THE EQUATION OF A LINE

$ax + by = c$	standard form
$y - y_1 = m(x - x_1)$	point-slope form
$y = mx + b$	slope-intercept form

FINDING THE EQUATION OF A LINE

Type 1: Given *one point* and *the slope*

Example: Find the equation of a line that passes through the point $(-13,11)$ and has a slope equal to -7.

Solution: We start by stating that the equation of a line is $y = mx + b$.

Since we are given m, the equation becomes $y = -7x + b$. Now all we need to do is solve for b. In order to find b, we use the fact that the line passes through $(-13,11)$.

Substituting this point into $y = -7x + b$ yields:

$$11 = -7(-13) + b$$
$$11 = 91 + b$$
$$11 - 91 = b$$
$$b = -80$$

∴ The equation of the line becomes $y = -7x - 80$.

Type 2: Given *two points*

Example: Find the equation of a line that passes through the points $(5,-5)$ and $(2,7)$.

Solution: We start by stating that the equation of a line is $y = mx + b$.

First we need to find m. Since we are given two points we can use the following formula to find m:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the points $(5,-5)$ and $(2,7)$ into the above equation yields:

$$m = \frac{7 - (-5)}{2 - 5} = \frac{12}{-3} = -4$$

The equation becomes $y = -4x + b$. Now all we need to do is solve for b. In order to find b, we use the fact that the line passes through $(5,-5)$ and $(2,7)$. By picking one of the two points and repeating the steps in type 1, we find that $b = 15$.

∴ The equation of the line becomes $y = -4x + 15$. (Why?)

Type 3: Given *one point* and a *parallel line*

Example: Find the equation of a line that passes through the point $(7, -1)$ and is parallel to the line $3x - 5y = 1$.

Solution: We start by stating that the equation of a line is $y = mx + b$.

First we need to find m. Since we are given a parallel line, we can use the fact that the slopes of parallel line are equal. We need to find the slope of $3x - 5y = 1$.

$$3x - 5y = 1$$
$$-5y = -3x + 1$$
$$y = \frac{-3x}{-5} + \frac{1}{-5}$$
$$y = \frac{3}{5}x - \frac{1}{5}$$

From the above equation we find that $m_1 = \frac{3}{5}$. Since the slopes of parallel line are equal, we conclude that $m = m_1 = \frac{3}{5}$. The equation becomes $y = \frac{3}{5}x + b$.

Now all we need to do is solve for b. In order to find b, we use the fact that the line passes through $(7, -1)$. By repeating the steps in type 1, we find that $b = -\frac{26}{5}$.

∴ The equation of the line becomes $y = \frac{3}{5}x - \frac{26}{5}$. (Why?)

Type 4: Given *one point* and a *perpendicular line*

Example: Find the equation of a line that passes through the point $(4,4)$ and is perpendicular to the line $7x + 3y + 4 = 0$.

Solution: We start by stating that the equation of a line is $y = mx + b$.

First we need to find m. Since we are given a perpendicular line, we can use the fact the slopes of perpendicular lines have the following relationship $m \cdot m_1 = -1$.

If we find the slope of $7x + 3y + 4 = 0$, we can find the slope for $y = mx + b$. By repeating the steps in type 3, we find that $m_1 = -\frac{7}{3}$.

Now we can find m from $m \cdot m_1 = -1$.

$$m\left(-\frac{7}{3}\right) = -1$$
$$m = -1\left(-\frac{3}{7}\right)$$
$$m = \frac{3}{7}$$

The equation becomes $y = \frac{3}{7}x + b$. Now all we need to do is solve for b. In order to find b, we use the fact that the line passes through $(4,4)$. By repeating the steps in type 1, we find that $b = \frac{16}{7}$.

∴ The equation of the line becomes $y = \frac{3}{7}x + \frac{16}{7}$. (Why?)

<u>Type 5</u>: Given *one point* and an angle

Example: Find the equation of a line that passes through the point $(3,17)$ and the angle between the line and the x-axis is equal to $37°$.

Solution: We start by stating that the equation of a line is $y = mx + b$.

First we need to find m. Since we are given an angle, we can use the fact the slope of the line $m = \tan 37°$.

The equation becomes $y = \tan 37° x + b$. Now all we need to do is solve for b. In order to find b, we use the fact that the line passes through $(3,17)$. By repeating the steps in type 1, we find that $b = 17 - 3\tan 37°$.

$\therefore$ The equation of the line becomes $y = \tan 37° x + 17 - 3\tan 37°$. (Why?)

<u>Type 6</u>: Given *two lines* and *the slope*

Example: Find the equation of a line that passes through the point of intersection of $x + y = 7$ and $7x - 5y = 1$ if it has a slope equal to -1.

Solution: We start by stating that the equation of a line is $y = mx + b$.

Since we are given m, the equation becomes $y = -x + b$. Now all we need to do is solve for b. In order to find b, we use the fact that the line passes through the intersection of $x + y = 7$ and $7x - 5y = 1$. We need to find where they intersect.

$$\begin{cases} x + y = 7 \\ 7x - 5y = 1 \end{cases}$$

Multiplying the first equation by 5 yields:

$$\begin{cases} 5x + 5y = 35 \\ 7x - 5y = 1 \end{cases}$$

Adding the two equations yields:

$$12x = 36$$
$$x = 3$$

Substituting $x = 3$ into $x + y = 7$ yields:

$$3 + y = 7$$
$$y = 4$$

The point of intersection of $x + y = 7$ and $7x - 5y = 1$ is $(3,4)$. Now we can find b. By repeating the steps in type 1, we find that $b = 7$.

$\therefore$ The equation of the line becomes $y = -x + 7$. (Why?)